

New Approach to Sgr A* Problem

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Abstract

The hypothesis that radiation of Sgr A* is caused by accretion onto a supermassive compact object without the events horizon is studied. The main equations of the accretion and a relativistic equation of the transfer radiation are obtained. The synchrotron spectrum in the vicinity of the maximum is considered.

1 Introduction

An analysis of stars motion in the dynamic center of the Galaxy gives evidences for the existence of a supermassive ($2.6 \cdot 10^6 M_{\odot}$) compact object. (See reviews [1], [2]). There are three kinds of explanation of the observed peculiarity of the object:

1. The gas accretion onto the central object - a supermassive black hole.
2. The ejection of the magnetized plasma from the vicinity of the Schwarzschild radius of the above object.
3. The explanations are based on some hypotheses about another nature of the central objects (neutrino ball, boson stars).

In the present paper we begin an investigation of the assumption that the radiation of Sgr A* is conditioned by a spherically symmetric accretion onto a supermassive compact object without the events horizon. The possibility of the existence of such a kind of objects follows [3], [4] from our gravitation equations [5]. This is a stable configuration of the Fermi-gas with radius R less than the Schwarzschild radius r_g (R is about $0.04 r_g$ for the mass $2.6 \cdot 10^6 M_{\odot}$) which unlike a black hole has no the events horizon.

It was shown earlier [6] that the accretion onto objects of this kind does not contradict the low observed bolometric luminosity of Sgr A* ($< 10^{37} \text{ erg/s}$).

The spectrum of the radiation near its maximum ($10^{13} \div 10^{14} \text{ Hz}$), that supposedly comes from the vicinity of the object surface, is obtained. It follows from the results that the analyzed model can be considered as one of the explanation of Sgr A* radiation.

2 Accretion equations

In view of the used bimetric gravitation equations [5] a remote observer can consider accretion on the object as a process in Pseudo-Euclidean space - time. Euler's equations of the relativistic hydrodynamics in flat space - time in the presence of a force-field $G^i(x)$ are of the form

$$c^{-2} \nabla_\alpha T^{\alpha i} = G^i, \quad (1)$$

where $T^{\alpha i}$ are the components of the energy-momentum tensor $T^{\alpha\beta}$ of a perfect fluid (Greek indexes run from 0 to 3 and Latin – from 1 to 3), ∇_α is the covariant derivative with respect to coordinates x^α in the used coordinate system.

In the case of the spherically symmetric accretion and spherical coordinates (t, r, φ, θ) we obtain the following differential equation for the radial component u of the 4-velocity u^α of gas

$$wuu' + (1 - u^2)P' = G^i/c^2, \quad (2)$$

where w is the mass-energy density of the gas, P is the pressure, the prime denotes a derivative with respect to r .

The field $G^i(x)$ consist of the force density of the radiation pressure P_{rad} , caused by the accretion, and the density F_g of the gravitational force.

It should be noted that according to [5] the maximal velocity of a test particle, falling free onto the object, does not exceed 0.4 of light velocity. Consequently, the radial component V of the 3-velocity satisfies the condition $V^2/c^2 \ll 1$.

As a result, the Euler equation of the spherically - symmetric accretion onto the compact object without the events horizon is given by

$$wVV' + c^2(P + P_{rad})' - F_g = 0. \quad (3)$$

In this equation

$$w = m_p c^2 n + P + \varepsilon, \quad (4)$$

where m_p is the proton mass, n is the density of particles, ε is the internal energy of the gas:

$$\varepsilon = \alpha nkT + 3P_{rad} + B^2/8\pi, \quad (5)$$

The first righr-hand term is the equation of state of an ionized gas, k is the Boltzman constant. (In this paper we set the constant $\alpha = 3$ at $10^5 K < T < 6 \cdot 10^9 K$, and $a = 9/2$ at $T > 6 \cdot 10^9 K$). The second term is the density energy of the radiation at the accretion, the third is the energy density of the magnetic field B , frozen in the accretioning plasma.

The density of the gravity force F_g can be found from the equation of the motion of the test particle in a given field. It is is of the form [5]

$$F_g = c^2 J_1 + (J_2 - 2J_3)V^2, \quad (6)$$

where

$$J_1 = \frac{C'}{2A}; \quad J_2 = \frac{A'}{2A}; \quad J_3 = \frac{C'}{2C}; \quad (7)$$

$$C = 1 - \frac{r_g}{f}; \quad A = \frac{f'^2}{C}; \quad f = (r_g^3 + r^3)^{1/3}. \quad (8)$$

The conservation of mass at the condition $V^2/c^2 \ll 1$ yields the equation

$$4\pi r^2 \rho V = const, \quad (9)$$

where the constant is the mass accretion rate $\dot{M} = dM/dt$.

The first equation of thermodynamics yields the equation

$$d\left(\frac{w}{n}\right) = Pd\left(\frac{1}{n}\right) + dQ, \quad (10)$$

where dQ is an increment of the heat per on a particle, caused by cooling and heating of the accretioning gas at the interval dr . We take into account cooling caused by sychrotron and bremsstrahlung radiation, and also by the converse Compton effect. According to [11] we suppose that contribution to dQ because of the bremsstrahlung (in CGS units) is

$$dQ_t = -1.4 \cdot 10^{-27} n \sqrt{T} V^{-1} dr, \quad (11)$$

and the one because of the sychrotron radiation and reverse Compton effect is given by

$$dQ_{s+k} = -(\xi_1 + \xi_2) k^2 T^2 V^{-1} dr, \quad (12)$$

where $\xi_1 = 2.37 \cdot 10^{-3} B^2$ and $\xi_2 = 3.97 \cdot 10^{-2} 3P_{rad}$.

The heating of the accretioning gas is caused by the dissipation of the magnetic field B as a consequence of maintaining the equality between the energy of the magnetic field B and the kinetic energy of the matter [7]

$$\frac{B^2}{8\pi} = \frac{m_p V^2}{2} \quad (13)$$

for all r , which is generally accepted. The contribution in dQ due to of it is given by

$$dQ_B = \frac{B^2}{8\pi n} \left(\frac{V'}{V} + \frac{2}{r} \right) dr. \quad (14)$$

Finally, we obtain the equation

$$\left(\frac{w}{n} \right)' = P \frac{n'}{n} + Q'_t + Q'_{s+k} + Q_B. \quad (15)$$

The simultaneous solution of the above equations for a given value of \dot{M} and magnitudes n , V and T at infinity allows us to find the functions $n(r)$, $V(r)$ and $T(r)$, which used for finding the spectrum of radiation.

3 Relativistic transfer equation

The weakest point of the existing models of Sgr A* is not rigorous enough consideration of the transfer radiation. In the equation an influence of gravitation on the frequency of the photon and its motion must be taken into account. Only the relativistic transfer equation can be used for this purpose. It is a relativistic Boltzman equations for photon gas [8], [9], [10]. We assume that in the spherically symmetric field the distribution function \mathcal{F} is

the function of the radial distance from the center r , of the frequency ν and the photon direction which can be defined by the cosine of the horizontal angle $-\mu$. (The spherical coordinate system is used). The relativistic Boltzman equation is given by

$$\frac{d\mathcal{F}}{ds} = St(\mathcal{F}), \quad (16)$$

where $d\mathcal{F}/ds$ is the 4-trajectory $x^\alpha(s)$ derivative in space-time with the metric differential form ds and the right-hand member is a relativistic collisions integral. By using for photons t (or x^0) as a parameter along 4-trajectory we arrive at the transfer equation in the form

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \mathbf{n} \nabla + \frac{d\mu}{dx^0} \frac{\partial}{\partial \mu} + \frac{d\nu}{dx^0} \frac{\partial}{\partial \nu} \right) \mathcal{F} = St(\mathcal{F}), \quad (17)$$

where the magnitudes $d\mu/dx^0$ and $d\nu/dx^0$ must be found from our equations of the photon motion in the gravitation field [3]. The collisions integral is of the form

$$\chi (S/\beta - \mathcal{F}), \quad (18)$$

where χ is the absorption coefficient, η is the emissivity, $S = \eta/\chi$ is the source function and $\beta = h^4 \nu^3 / c^2$. In this paper we do not take into account light diffusion. An intensity I of the radiation is related to \mathcal{F} as $I = \beta \mathcal{F}$.

4 Spectrum of the synchrotron radiation of Sgr A*

We have used the characteristic method [9], [10] for the solution of the transfer equation. Since photon's trajectories are the characteristics of the partial differential equation (17), they are reduced to ordinary differential equations along these trajectories. In our case the equations are

$$\frac{d\mathcal{F}}{dr} = \frac{c}{v} \chi \left(\frac{S}{\beta} - \mathcal{F} \right), \quad (19)$$

where v is the radial photon velocity.

According to [5] in the spherically - symmetric field

$$v = c \sqrt{\frac{C}{A} \left(1 - C \frac{b^2}{f^2} \right)}, \quad (20)$$

where b is the impact parameter of photon.

For numerical estimates we set emissivity [11]

$$\eta = c_3 B n F(X), \quad (21)$$

where in CGS units $c_3 = 1.87 \cdot 10^{-23}$, B is the magnetic field, n is the electron density as the function of r ,

$$F(X) = X \int_X^\infty K_{5/3}(z) dz, \quad (22)$$

$K_{5/3}(z)$ is the modified Bessel function of the second kind, $X = \nu/\nu_c$, $\nu_c = c_1 B (kT)^2$, $c_1 = 6.27 \cdot 10^{18}$, T is the temperature as a function of r , and k is Boltzman's constant.

The absorption coefficient is approximately

$$\chi = \frac{c_4 B^{3.2} n}{\nu_c^{5/2}} K_{5/3}(X), \quad (23)$$

where $c_4 = 4.20 \cdot 10^7$.

The function $n(r)$ is given by

$$n = \frac{\dot{M}}{4\pi m_p V r^2}. \quad (24)$$

Let $\mathcal{F}_\nu(b)$ is the solution $\mathcal{F}_\nu(r, b)$ of the differential equation (19) for a given b at $r \rightarrow \infty$. Then for a distant observer the luminosity at the frequency ν is given by

$$L_\nu = 8\pi^2 \int_0^\infty \beta \mathcal{F}_\nu(b) b db, \quad (25)$$

For the correct solution of eq.(19) it is essential that there is three types of photons trajectories in the spherically-symmetric gravitation field in view of the used gravitation equations. It can be seen from fig. 1. It shows the geometrical locus of the points where the radial photon velocities are equal to zero which is given by the equation

$$b = \frac{f}{\sqrt{C}} \quad (26)$$

The minimal value of b is $b_{cr} = 3\sqrt{3}r_g/2$. It occurs at the distance from the center $r_{cr} = \sqrt[3]{19}r_g/2$. The photons whose impact parameter $b < b_{cr}$

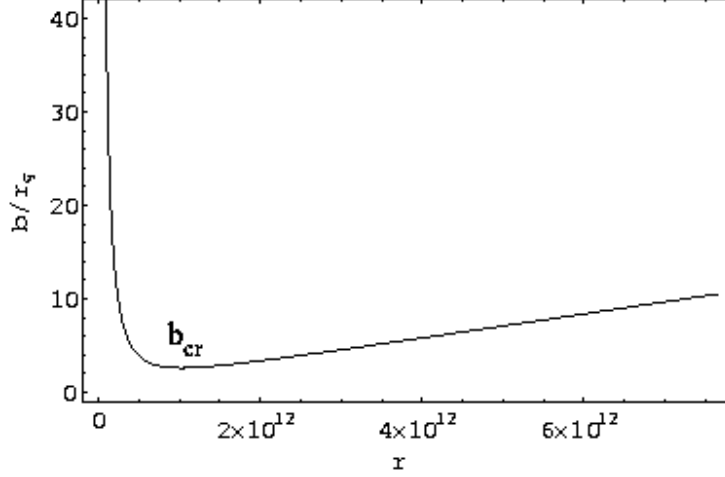


Figure 1: The dependence b of r for null-points of the function $v(r)$.

freely extend from the object surface to infinity. The photons with $b > b_{cr}$ can extend to infinity only if their trajectories begin at $r > r_{cr}$. For a given b the corresponding magnitude of r can be found from eq.(26)

The differential equation for photon trajectories with $b < b_{cr}$ were integrated at the edge condition $\mathcal{F}(R) = 0$. The distribution function \mathcal{F} at the points of the curve in fig. 1 at $r > r_{cr}$ was found by solution of differential equation (19) by used the end condition $\mathcal{F} = 0$ at infinity. Similarly to [10] these magnitudes were used as end conditions for the solution of the (19) to find an output radiation.

Fig.2 shows the spectrum of the synchrotron radiation in the band $(10^{10} \div 2 \cdot 10^{13})$ Hz.

In the studied frequencies band the functions $\mathcal{F}_{\infty}(b)$ are weakly depending on b up to some maximal magnitude b_{max} , and after that quickly decreases. In fig. 3 the dependence of b_{max} on the frequency is shown.

A comparison (2) and (3) with the observation data [1], [2] shows that the results do not contradict observations and, therefore, can be a basis for a deeper investigation.

5 Conclusion

To clear up question about the nature of Sgr A* is issue of the day since the problem is the nature of the supermassive compact objects. Since 1992 a number of models have been created that are able to explain Sgr A* spectrum.

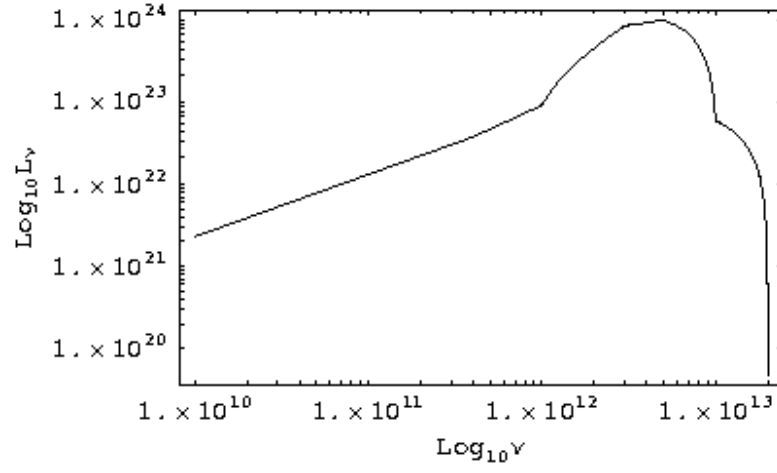


Figure 2: The Synchrotron spectrum of Sgr A* near the maximum.

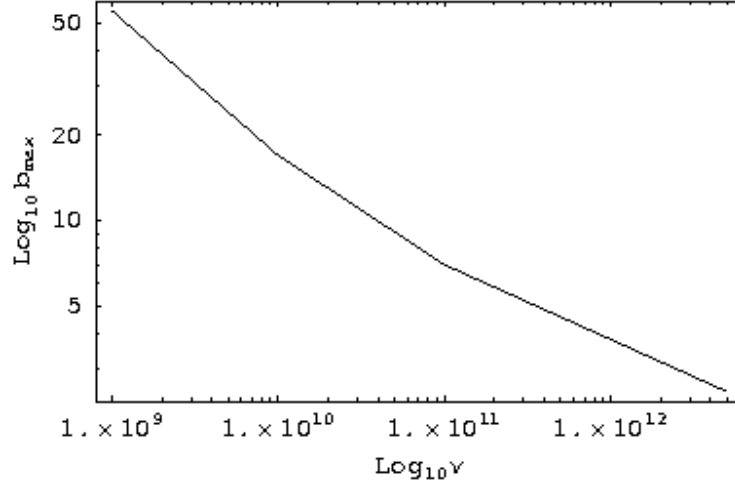


Figure 3: The maximal value of the impact parameter vs the frequency.

It is the possibility of an ambiguous explanation of the spectrum that means that we need for more rigorous methods of the calculation of the Sgr A* spectrum. It is possible only by a simultaneous solution of the relativistic hydrodynamics and transfer equations.

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